Galactic tide in a noninertial frame of reference

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Abstract Equation of motion and the vector of perturbing acceleration (force) for the galactic tide in a noninertial frame of reference is derived. The noninertial reference frame is rotating with a fixed angular velocity $\boldsymbol{\omega} = - \omega_0 \, \hat{\mathbf{z}}$ with respect to the inertial frame of reference of the Galaxy. $\boldsymbol{\omega}$ is the angular velocity of the solar rotation (rotation of the Local Standard of Rest) around the galactic center, the unit vector $\hat{\mathbf{z}}$ is oriented toward the north pole of the Galaxy: the Sun is always situated in the plane $y' = 0$ ($x'-z'$-plane). The equation of motion can be applied to the dynamics of the Oort cloud of comets. Relations for calculation of the osculating orbital elements are presented and a new integral of motion is derived for the conventional approach in modelling of the effect of the galactic tidal field.

Keywords Oort cloud · Comets · Equation of motion

1 Introduction

Global galactic gravitational field influences motion of a comet in the Oort cloud in the form of the galactic tide. The effect of the galactic tide was physicaly treated by Klačka (2009).

This paper presents the equation of motion in a noninertial frame of reference S' (primed quantities). The frame of reference S' is rotating with a constant angular velocity $\boldsymbol{\omega} = - \omega_0 \, \hat{\mathbf{z}}$ around the galactic center with respect to the galactic inertial frame of reference S (it's origin is at the center of the Galaxy, $x-$ and $y-$axes lie in the galactic equatorial plane). The unit vector $\hat{\mathbf{z}}$ is oriented toward the north pole of the Galaxy, the plane $z = z' = 0$ is the plane of the galactic equator. The Sun is always situated in the plane $y' = 0$ with respect to the noninertial reference frame S'.

Both sides (left-hand and right-hand sides) of the derived equation of motion contain the quantities measured in the noninertial frame of reference. This is consistent with physics and not the only difference from the conventional approach, as it is presented by Heisler and Tremaine (1986, Eqs. 4 and 6), or, as for the newest papers, e.g., by Dybczynski et al. (2008). Dybczynski et al. (2008) write on p. 347: "The

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$Ox'y'z'-$coordinate system rotates on the large timescale of our simulation." "In the modified heliocentric galactic $Ox'y'z'-$coordinate system, in which the $x'-$axis is orientated outward from the Galactic centre and the $z'-$axis is orientated toward the South Galactic pole, the vector of perturbing force can be written as $\mathbf{F} = (K_x x', K_y y', K_z z')$, where x', y', z' are the rectangular coordinates of the given TP (test particle) in the modified heliocentric galactic-coordinate-system, and $K_x = (A - B)(3A + B)$, $K_y = -(A-B)^2$, $K_z = -[4\pi k^2 \rho_{GM} - 2(B^2 - A^2)]$." Our equation of motion contains physical terms connected with the noninertiality of the reference frame. The corresponding part of the equation of motion is not consistent with the above cited description. Moreover, our equation of motion contains new gravitational terms.

2 Equation of motion in an inertial frame of reference

We are interested in motion of a comet with respect to the Sun, if gravity of the Sun and Galaxy act. Currently, the Sun is situated $R_0 = 8$ *kpc* from the center of the Galaxy and 30 *pc* above the galactic equatorial plane ($Z_0 = 30$ *pc*). Besides rotational motion with the speed $(A - B)\ R_0$ the Sun moves with the speed 7.3 *km/s* in the direction normal to the galactic plane. Positional vector of the comet with respect to the Sun is $\mathbf{r} = (\xi, \eta, \zeta)$ in the inertial frame of reference S. Equation of motion in the inertial frame of reference yields

$$
\begin{aligned}
\frac{d^2\xi}{dt^2} &= -\ \frac{GM_\odot}{r^3}\ \xi\ +\ (A-B)\,[A+B+2A\cos(2\ \omega_0 t)]\ \xi \\
&\quad -\ 2A(A-B)\sin(2\ \omega_0 t)\ \eta \\
&\quad +\ 2\ (A-B)^2\ (\mathit{\Gamma}_1\ -\ \mathit{\Gamma}_2\ Z_0^2)\ R_0\ Z_0\ \cos(\omega_0 t)\ \zeta\ , \\
\frac{d^2\eta}{dt^2} &= -\ \frac{GM_\odot}{r^3}\ \eta\ -\ 2A(A-B)\sin(2\ \omega_0 t)\ \xi \\
&\quad +\ (A-B)\,[A+B-2A\cos(2\ \omega_0 t)]\ \eta \\
&\quad -\ 2\ (A-B)^2\ (\mathit{\Gamma}_1\ -\ \mathit{\Gamma}_2\ Z_0^2)\ R_0\ Z_0\ \sin(\omega_0 t)\ \zeta\ , \\
\frac{d^2\zeta}{dt^2} &= -\ \frac{GM_\odot}{r^3}\ \zeta\ -\ \left[4\ \pi\ G\ \varrho\ +\ 2\left(A^2\ -\ B^2\right)\right]\ \zeta \\
&\quad -\ 4\ \pi\ G\ \varrho'\ Z_0\,[\xi\ \cos(\omega_0 t)\ -\ \eta\ \sin(\omega_0 t)]\ , \\
\frac{d^2 Z_0}{dt^2} &= -\ \left[4\ \pi\ G\ \varrho\ +\ 2\left(A^2\ -\ B^2\right)\right]\ Z_0\ , \\
r &= \sqrt{\xi^2\ +\ \eta^2\ +\ \zeta^2}\ , \\
\omega_0 &= A\ -\ B\ ,
\end{aligned}
\tag{1}
$$

where G is the gravitational constant, $M_\odot$ is the mass of the Sun and the numerical values of the other relevant quantities are

$$
\begin{aligned}
A &= 14.2\ km\ s^{-1}\ kpc^{-1}\ , \\
B &= -\ 12.4\ km\ s^{-1}\ kpc^{-1}\ , \\
\mathit{\Gamma}_1 &= 0.124\ kpc^{-2}\ , \\
\mathit{\Gamma}_2 &= 1.586\ kpc^{-4}\ , \\
\varrho &= 0.130\ M_\odot\ pc^{-3}\ , \\
\varrho' &= -\ 0.037\ M_\odot\ pc^{-3}\ kpc^{-1}\ ,
\end{aligned}
\tag{2}
$$

see Eqs. (26)-(27) in Klačka (2009). If one wants to use other values of the Oort constants A and B, then he can use the following equation for calculation of mass density in the neighborhood of the Sun:

$$
\begin{aligned}
\varrho &= \varrho_{disk} \ + \ \varrho_{halo} \ , \\
\varrho_{disk} &= 0.126 \ M_{\odot} \ pc^{-3} \ , \\
\varrho_{halo} &= (4 \pi G)^{-1} [X(Galaxy) + X(disk) + X(bulge)] \ , \\
X(Galaxy) &\equiv - (A - B) \times (A + 3B) \\
X(disk) &= - \ 396.90 \ km^{2} \ s^{-2} \ kpc^{-2} \ , \\
X(bulge) &= - \ 0.65 \ km^{2} \ s^{-2} \ kpc^{-2} \ .
\end{aligned} \tag{3}
$$

Eqs. (22) of Klačka (2009) can be used.

2.1 Vector of perturbing acceleration

On the basis of Eqs. (1) we can state that dominant force is represented by gravity of the Sun and the perturbing force is given by gravity of the Galaxy. The perturbing acceleration due to the action of the Galaxy is

$$
\begin{aligned}
\mathbf{F} &= F_x \ \hat{\mathbf{x}} \ + \ F_y \ \hat{\mathbf{y}} \ + \ F_z \ \hat{\mathbf{z}} \ , \\
F_x &= (A - B) \left[A + B + 2A \cos{(2 \ \omega_0 t)} \right] \ \xi \\
&\quad - \ 2A(A - B) \sin{(2 \ \omega_0 t)} \ \eta \\
&\quad + \ 2 \ (A - B)^2 \ (\mathit{\Gamma}_1 \ - \ \mathit{\Gamma}_2 \ Z_0^2) \ R_0 \ Z_0 \ \cos{(\omega_0 t)} \ \zeta \ , \\
F_y &= - \ 2A(A - B) \sin{(2 \ \omega_0 t)} \ \xi \\
&\quad + \ (A - B) \left[A + B - 2A \cos{(2 \ \omega_0 t)} \right] \ \eta \\
&\quad - \ 2 \ (A - B)^2 \ (\mathit{\Gamma}_1 \ - \ \mathit{\Gamma}_2 \ Z_0^2) \ R_0 \ Z_0 \ \sin{(\omega_0 t)} \ \zeta \ , \\
F_z &= - \ \left[4 \ \pi \ G \ \varrho \ + \ 2 \left(A^2 \ - \ B^2 \right) \right] \ \zeta \\
&\quad - \ 4 \ \pi \ G \ \varrho' \ Z_0 \left[\xi \ \cos{(\omega_0 t)} \ - \ \eta \ \sin{(\omega_0 t)} \right] \ .
\end{aligned} \tag{4}
$$

The perturbing acceleration ceases to exist when $A \equiv B \equiv \varrho \equiv 0$.

3 Equation of motion in a noninertial frame of reference

We are interested in the motion of a comet with respect to the Sun in the noninertial frame of reference S'. The noninertial frame of reference is defined as the frame rotating with a constant angular velocity $\boldsymbol{\omega} = \omega \ \hat{\mathbf{z}}$ around the galactic center with respect to the galactic inertial frame of reference S (it's origin is at the center of the Galaxy, $x-$ and $y-$axes lie in the galactic equatorial plane). The unit vector $\hat{\mathbf{z}}$ is oriented toward the north pole of the Galaxy, the plane $z = z' = 0$ is the plane of the galactic equator. The Sun is moves in the plane $y' = 0$ with respect to the noninertial reference frame S'. The Sun's galactocentric position vector is

$$
\mathbf{R}_0 = R_0 \ \cos(- \ \omega_0 \ t) \ \hat{\mathbf{x}} \ + \ R_0 \ \sin(- \ \omega_0 \ t) \ \hat{\mathbf{y}} \ + \ Z_0 \ \hat{\mathbf{z}} \ , \tag{5}
$$

in the inertial frame of reference and

$$\mathbf{R}_0' = R_0 \ \hat{\mathbf{x}}' \ + \ Z_0 \ \hat{\mathbf{z}}' \ , \tag{6}$$

in the noninertial frame of reference, since $\mathbf{R}_0' = \mathbf{R}_0$. The sign minus at ω_0 denotes negative (clockwise) orientation/direction of the motion of the Sun, $\boldsymbol{\omega} = \omega \ \hat{\mathbf{z}} = - \ \omega_0 \ \hat{\mathbf{z}}$, $R_0 = 8 \ kpc$.

3.1 Transformations

The unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ form an orthonormal basis and the right-handed system in the inertial frame of reference S. The same holds for the primed unit vectors of the rotating noninertial system S', and

$$\begin{aligned}
\hat{\mathbf{x}}' &= \cos(\omega \ t) \ \hat{\mathbf{x}} \ + \ \sin(\omega \ t) \ \hat{\mathbf{y}} \ , \\
\hat{\mathbf{y}}' &= - \ \sin(\omega \ t) \ \hat{\mathbf{x}} \ + \ \cos(\omega \ t) \ \hat{\mathbf{y}} \ , \\
\hat{\mathbf{z}}' &= \hat{\mathbf{z}} \ .
\end{aligned} \tag{7}$$

The inverse relations are

$$\begin{aligned}
\hat{\mathbf{x}} &= \cos(\omega \ t) \ \hat{\mathbf{x}}' \ - \ \sin(\omega \ t) \ \hat{\mathbf{y}}' \ , \\
\hat{\mathbf{y}} &= \sin(\omega \ t) \ \hat{\mathbf{x}}' \ + \ \cos(\omega \ t) \ \hat{\mathbf{y}}' \ , \\
\hat{\mathbf{z}} &= \hat{\mathbf{z}}' \ .
\end{aligned} \tag{8}$$

Let us consider a point mass with position vector $\mathbf{r} = \mathbf{r}'$. We have $\mathbf{r} = (x, \, y, \, z)$ and $\mathbf{r}' = (x', \, y', \, z')$, or, $\mathbf{r} = x \ \hat{\mathbf{x}} + y \ \hat{\mathbf{y}} + z \ \hat{\mathbf{z}}$, $\mathbf{r}' = x' \ \hat{\mathbf{x}}' + y' \ \hat{\mathbf{y}}' + z' \ \hat{\mathbf{z}}'$. The components in the inertial and noninertial reference frames are related through the relations

$$\begin{aligned}
x' &= x \ \cos(\omega \ t) \ + \ y \ \sin(\omega \ t) \ , \\
y' &= - \ x \ \sin(\omega \ t) \ + \ y \ \cos(\omega \ t) \ , \\
z' &= z \ .
\end{aligned} \tag{9}$$

The orthogonal transformation given by Eqs. (9) immediately offers the inverse transformation:

$$\begin{aligned}
x &= x' \ \cos(\omega \ t) \ - \ y' \ \sin(\omega \ t) \ , \\
y &= x' \ \sin(\omega \ t) \ + \ y' \ \cos(\omega \ t) \ , \\
z &= z' \ .
\end{aligned} \tag{10}$$

The velocity with respect to the system S' is $\mathbf{v}' \equiv d'\mathbf{r}'/dt \equiv (dx'/dt) \ \hat{\mathbf{x}}' + (dy'/dt) \ \hat{\mathbf{y}}' + (dz'/dt) \ \hat{\mathbf{z}}'$. The relation $\mathbf{v}' = (dx'/dt) \ \hat{\mathbf{x}}' + (dy'/dt) \ \hat{\mathbf{y}}' + (dz'/dt) \ \hat{\mathbf{z}}'$ and Eqs. (8) yield

$$\begin{aligned}
(\mathbf{v}')_x &\equiv \mathbf{v}' \cdot \hat{\mathbf{x}} = \frac{dx'}{dt} \ \cos(\omega \ t) \ - \ \frac{dy'}{dt} \ \sin(\omega \ t) \ , \\
(\mathbf{v}')_y &\equiv \mathbf{v}' \cdot \hat{\mathbf{y}} = \frac{dx'}{dt} \ \sin(\omega \ t) \ + \ \frac{dy'}{dt} \ \cos(\omega \ t) \ , \\
(\mathbf{v}')_z &\equiv \mathbf{v}' \cdot \hat{\mathbf{z}} = \frac{dz'}{dt} \ .
\end{aligned} \tag{11}$$

It can be said that Eqs. (11) hold on the basis of vectorial transformation defined by Eqs. (10), because any vector in the rotating frame must project onto $x-$, $y-$ and $z-$ axes in the same way as any other vector (Kittel et al. 1965, p. 85). As for the acceleration vector, we have $\mathbf{a}' \equiv d'\mathbf{v}'/dt = d'\ ^2\mathbf{r}'/dt^2$ with respect to the system S', and

$$
\begin{aligned}
(\mathbf{a}')_x &\equiv \mathbf{a}' \cdot \hat{\mathbf{x}} = \frac{d^2x'}{dt^2}\ \cos(\omega\ t)\ -\ \frac{d^2y'}{dt^2}\ \sin(\omega\ t)\ , \\
(\mathbf{a}')_y &\equiv \mathbf{a}' \cdot \hat{\mathbf{y}} = \frac{d^2x'}{dt^2}\ \sin(\omega\ t)\ +\ \frac{d^2y'}{dt^2}\ \cos(\omega\ t)\ , \\
(\mathbf{a}')_z &\equiv \mathbf{a}' \cdot \hat{\mathbf{z}} = \frac{d^2z'}{dt^2}\ .
\end{aligned} \tag{12}
$$

Similarly,

$$
\begin{aligned}
(\boldsymbol{\omega} \times \mathbf{v}')_x &= -\ \omega\ (\mathbf{v}')_y = -\ \omega\ \left\{ \frac{dx'}{dt}\ \sin(\omega\ t)\ +\ \frac{dy'}{dt}\ \cos(\omega\ t) \right\}\ , \\
(\boldsymbol{\omega} \times \mathbf{v}')_y &= +\ \omega\ (\mathbf{v}')_x = +\ \omega\ \left\{ \frac{dx'}{dt}\ \cos(\omega\ t)\ -\ \frac{dy'}{dt}\ \sin(\omega\ t) \right\}
\end{aligned} \tag{13}
$$

and

$$
\begin{aligned}
[\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')]_x &= -\ \omega^2\ (\mathbf{r}')_x = -\ \omega^2\ \left\{ x' \cos(\omega\ t) - y' \sin(\omega\ t) \right\}\ , \\
\left[\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')\right]_y &= -\ \omega^2\ (\mathbf{r}')_y = -\ \omega^2\ \left\{ x' \sin(\omega\ t) + y' \cos(\omega\ t) \right\}\ .
\end{aligned} \tag{14}
$$

On the basis of Eqs. (7)-(14) we can write

$$
\mathbf{a} = \mathbf{a}'\ +\ 2\ \boldsymbol{\omega} \times \mathbf{v}'\ +\ \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')\ , \tag{15}
$$

for accelerations in the inertial (unprimed quantities) and noninertial (primed quantities) reference frames (see, e.g., Kittel et al. 1965, pp. 83-85).

3.2 Galactic tide in the noninertial frame of reference

On the basis of Eqs. (10)-(15), we can write for the $x-$ ($\xi-$) component of the acceleration given by Eqs. (1):

$$
\begin{aligned}
LHS_x &= RHS_x\ , \\
LHS_x &\equiv -\ \frac{GM_\odot}{r^3}\ \left[\xi' \cos(\omega_0 t)\ \ +\ \eta'\ \sin(\omega_0 t)\right] \\
&\quad +\ (A-B)\left[(A+B) + 2A\cos(2\omega_0 t)\right]\left[\xi' \cos(\omega_0 t)\ \ +\ \eta'\ \sin(\omega_0 t)\right] \\
&\quad -\ 2A(A-B)\ \sin(2\omega_0 t)\left[-\ \xi' \sin(\omega_0 t)\ \ +\ \eta'\ \cos(\omega_0 t)\right] \\
&\quad +\ 2\ (A-B)^2\ (\mathit{\Gamma}_1\ -\ \mathit{\Gamma}_2\ Z_0^2)\ R_0\ Z_0\ \cos(\omega_0 t)\ \zeta'\ , \\
RHS_x &\equiv \frac{d^2\xi'}{dt^2}\ \cos(\omega_0 t)\ +\ \frac{d^2\eta'}{dt^2}\ \sin(\omega_0 t) \\
&\quad +\ 2\omega_0 \left[-\ \frac{d\xi'}{dt}\ \sin(\omega_0 t)\ +\ \frac{d\eta'}{dt}\ \cos(\omega_0 t)\right] \\
&\quad -\ \omega_0^2\left[\xi'\ \cos(\omega_0 t) + \eta'\ \sin(\omega_0 t)\right]
\end{aligned} \tag{16}
$$

or, after some algebra,

$$
\begin{aligned}
LHS_x &= RHS_x \ , \\
LHS_x &\equiv - \ \frac{GM_\odot}{r^3} \ \left[\xi' \cos{(\omega_0 t)} \ + \ \eta' \ \sin{(\omega_0 t)}\right] \\
&\quad + (A-B) \left[(3A+B) \ \xi' \cos{(\omega_0 t)} \ - \ (A-B) \ \eta' \ \sin{(\omega_0 t)}\right] \\
&\quad + 2 \ (A-B)^2 \ (\mathit{\Gamma}_1 \ - \ \mathit{\Gamma}_2 \ Z_0^2) \ R_0 \ Z_0 \ \cos{(\omega_0 t)} \ \zeta' \ , \\
RHS_x &\equiv \frac{d^2\xi'}{dt^2} \ \cos{(\omega_0 t)} \ + \ \frac{d^2\eta'}{dt^2} \ \sin{(\omega_0 t)} \\
&\quad + 2\omega_0 \left[- \ \frac{d\xi'}{dt} \ \sin{(\omega_0 t)} \ + \ \frac{d\eta'}{dt} \ \cos{(\omega_0 t)}\right] \\
&\quad - \omega_0^2 \left[\xi' \ \cos{(\omega_0 t)} + \eta' \ \sin{(\omega_0 t)}\right] \ .
\end{aligned} \tag{17}
$$

On the basis of Eqs. (10)-(15), we can write for the $y-$ ($\eta-$) component of the acceleration given by Eqs. (1):

$$
\begin{aligned}
LHS_y &= RHS_y \ , \\
LHS_y &\equiv - \ \frac{GM_\odot}{r^3} \ \left[- \ \xi' \sin{(\omega_0 t)} \ + \ \eta' \ \cos{(\omega_0 t)}\right] \\
&\quad - 2A(A-B) \ \sin{(2\omega_0 t)} \left[\xi' \cos{(\omega_0 t)} \ + \ \eta' \ \sin{(\omega_0 t)}\right] \\
&\quad + (A-B)[A+B-2A \ \cos{(2\omega_0 t)}] \left[- \ \xi' \sin{(\omega_0 t)} \ + \ \eta' \ \cos{(\omega_0 t)}\right] \\
&\quad - 2 \ (A-B)^2 \ (\mathit{\Gamma}_1 \ - \ \mathit{\Gamma}_2 \ Z_0^2) \ R_0 \ Z_0 \ \sin{(\omega_0 t)} \ \zeta' \ , \\
RHS_y &\equiv - \ \frac{d^2\xi'}{dt^2} \ \sin{(\omega_0 t)} \ + \ \frac{d^2\eta'}{dt^2} \ \cos{(\omega_0 t)} \\
&\quad - 2\omega_0 \left[\frac{d\xi'}{dt} \ \cos{(\omega_0 t)} \ + \ \frac{d\eta'}{dt} \ \sin{(\omega_0 t)}\right] \\
&\quad - \omega_0^2 \left[- \ \xi' \ \sin{(\omega_0 t)} + \eta' \ \cos{(\omega_0 t)}\right]
\end{aligned} \tag{18}
$$

or, after some algebra,

$$
\begin{aligned}
LHS_y &= RHS_y \ , \\
LHS_y &\equiv - \ \frac{GM_\odot}{r^3} \ \left[- \ \xi' \sin{(\omega_0 t)} \ + \ \eta' \ \cos{(\omega_0 t)}\right] \\
&\quad - (A-B) \left[(3A+B) \ \xi' \sin{(\omega_0 t)} \ + \ (A-B) \ \eta' \ \cos{(\omega_0 t)}\right] \\
&\quad - 2 \ (A-B)^2 \ (\mathit{\Gamma}_1 \ - \ \mathit{\Gamma}_2 \ Z_0^2) \ R_0 \ Z_0 \ \sin{(\omega_0 t)} \ \zeta' \ , \\
RHS_y &\equiv - \ \frac{d^2\xi'}{dt^2} \ \sin{(\omega_0 t)} \ + \ \frac{d^2\eta'}{dt^2} \ \cos{(\omega_0 t)} \\
&\quad - 2\omega_0 \left[\frac{d\xi'}{dt} \ \cos{(\omega_0 t)} \ + \ \frac{d\eta'}{dt} \ \sin{(\omega_0 t)}\right] \\
&\quad - \omega_0^2 \left[- \ \xi' \ \sin{(\omega_0 t)} + \eta' \ \cos{(\omega_0 t)}\right] \ .
\end{aligned} \tag{19}
$$

Eqs. (17) and (19) yield

$$
\begin{aligned}
\frac{d^2\xi'}{dt^2} &= - \ \frac{GM_\odot}{r^3} \ \xi' \ + \ (A-B)(3A+B) \ \xi' \\
&\quad + 2 \ (A-B)^2 \ (\mathit{\Gamma}_1 \ - \ \mathit{\Gamma}_2 \ Z_0^2) \ R_0 \ Z_0 \ \zeta'
\end{aligned}
$$

$$- 2\omega_0 \frac{d\eta'}{dt} + \omega_0^2 \, \xi' ,$$

$$\frac{d^2\eta'}{dt^2} = - \frac{GM_\odot}{r^3} \, \eta' - (A - B)^2 \, \eta' + 2\omega_0 \frac{d\xi'}{dt} + \omega_0^2 \, \eta' . \tag{20}$$

Finally, Eqs. (1) and (20) yield

$$\begin{aligned}
\frac{d^2\xi'}{dt^2} &= - \frac{GM_\odot}{r^3} \, \xi' + 4 \, A \, (A - B) \, \xi' \\
&\quad + 2 \, (A - B)^2 \, (\mathit{\Gamma}_1 - \mathit{\Gamma}_2 \, Z_0^2) \, R_0 \, Z_0 \, \zeta' - 2 \, (A - B) \, \frac{d\eta'}{dt} , \\
\frac{d^2\eta'}{dt^2} &= - \frac{GM_\odot}{r^3} \, \eta' + 2 \, (A - B) \, \frac{d\xi'}{dt} , \\
\frac{d^2\zeta'}{dt^2} &= - \frac{GM_\odot}{r^3} \, \zeta' - \left[4 \, \pi \, G \, \varrho + 2 \, \left(A^2 - B^2 \right) \right] \, \zeta' \\
&\quad - 4 \, \pi \, G \, \varrho' \, Z_0 \, \xi' , \\
\frac{d^2 Z_0}{dt^2} &= - \left[4 \, \pi \, G \, \varrho + 2 \, \left(A^2 - B^2 \right) \right] \, Z_0 , \\
r &= \sqrt{\xi'^{\,2} + \eta'^{\,2} + \zeta'^{\,2}} .
\end{aligned} \tag{21}$$

3.3 Vector of perturbing acceleration

The perturbing force is given by the effect of Galaxy, according to Sec. 2.1. This effect is represented by the Oort constants A, B and mass density ϱ.

The vector of the perturbing acceleration acting on the comet is

$$\begin{aligned}
\mathbf{F}' &= (\mathbf{F}')_{\xi'} \, \hat{\boldsymbol{\xi}}' + (\mathbf{F}')_{\eta'} \, \hat{\boldsymbol{\eta}}' + (\mathbf{F}')_{\zeta'} \, \hat{\boldsymbol{\zeta}}' , \\
(\mathbf{F}')_{\xi'} &= 2 \, (A - B) \, \left\{ 2 \, A \, \xi' + (A - B) \, (\mathit{\Gamma}_1 - \mathit{\Gamma}_2 \, Z_0^2) \, R_0 \, Z_0 \, \zeta' - \frac{d\eta'}{dt} \right\} , \\
(\mathbf{F}')_{\eta'} &= 2 \, (A - B) \, \frac{d\xi'}{dt} , \\
(\mathbf{F}')_{\zeta'} &= - \left[4 \, \pi \, G \, \varrho + 2 \, \left(A^2 - B^2 \right) \right] \, \zeta' - 4 \, \pi \, G \, \varrho' \, Z_0 \, \xi' ,
\end{aligned} \tag{22}$$

if Eqs. (21) are taken into account.

The effect of Galaxy is turned off when $A = B = \varrho = \varrho' = 0$. $\mathbf{F}' = 0$ and the two-body problem exists in this case. This is consistent with Sec. 2.1.

4 Discussion

We want to concentrate on obtaining the evolution of osculating orbital elements on the basis of solution of Eqs. (21).

4.1 Osculating orbital elements

Eqs. (21) offer the values of coordinates ξ', η', ζ' and velocities $d\xi'/dt$, $d\eta'/dt$ and $d\zeta'/dt$ for a time t. We have to obtain the values in the inertial frame of reference S. We need ξ, η, ζ, $d\xi/dt$, $d\eta/dt$ and $d\zeta/dt$ for the time t.

On the basis of Eq. (20) we obtain

$$\begin{aligned} \xi &= \xi' \ \cos(\omega_0 \ t) \ + \ \eta' \ \sin(\omega_0 \ t) \ , \\ \eta &= - \ \xi' \ \sin(\omega_0 \ t) \ + \ \eta' \ \cos(\omega_0 \ t) \ , \\ \zeta &= \zeta' \ . \end{aligned} \tag{23}$$

The velocity components $d\xi/dt$, $d\eta/dt$ and $d\zeta/dt$ can be obtained from Eq. (23):

$$\begin{aligned} \frac{d\xi}{dt} &= \frac{d\xi'}{dt} \ \cos(\omega_0 \ t) \ + \ \frac{d\eta'}{dt} \ \sin(\omega_0 \ t) \\ &\quad - \omega_0 \ \xi' \ \sin(\omega_0 \ t) \ + \ \omega_0 \ \eta' \ \cos(\omega_0 \ t) \ , \\ \frac{d\eta}{dt} &= - \ \frac{d\xi'}{dt} \ \sin(\omega_0 \ t) \ + \ \frac{d\eta'}{dt} \ \cos(\omega_0 \ t) \\ &\quad - \omega_0 \ \xi' \ \cos(\omega_0 \ t) \ - \ \omega_0 \ \eta' \ \sin(\omega_0 \ t) \ , \\ \frac{d\zeta}{dt} &= \frac{d\zeta'}{dt} \ . \end{aligned} \tag{24}$$

Eqs. (24) are consistent with the relation

$$\mathbf{v} = \mathbf{v}' \ + \ \boldsymbol{\omega} \times \mathbf{r}' \ , \tag{25}$$

since $\boldsymbol{\omega} \times \mathbf{r}' = \omega \hat{\mathbf{z}}' \times (x' \ \hat{\mathbf{x}}' + y' \ \hat{\mathbf{y}}') = \omega \ (x' \ \hat{\mathbf{y}}' - y' \ \hat{\mathbf{x}}')$ and $(\ \boldsymbol{\omega} \times \mathbf{r}' \ ) \cdot \hat{\mathbf{x}} = \omega \ (x' \ \hat{\mathbf{y}}' \cdot \hat{\mathbf{x}} - y' \ \hat{\mathbf{x}}' \cdot \hat{\mathbf{x}} \ )$ and Eqs. (7) hold; similar consideration can be used also for $(\ \boldsymbol{\omega} \times \mathbf{r}' \ ) \cdot \hat{\mathbf{y}}$.

The evolution of the orbital elements is obtained on the basis of Eqs. (21), (23)-(24) and the relations presented by Klačka (2004) in his Eqs. (47) (the right-hand side of the last equation in Eqs. 47 must contain $1/e$ instead of 1). We can summarize the equations in the following form [osculating orbital elements: a – semi-major axis; e – eccentricity; i – inclination of the orbital plane to the reference plane – galactic equatorial plane; Ω – longitude of the ascending node; ω – the argument of pericenter/perihelion; Θ is the position angle of the particle on the orbit, when measured from the ascending node in the direction of the particle's motion, $\Theta = \omega + f$]:

$$\begin{aligned} \mathbf{r} &= (\xi, \eta, \zeta) \ , \quad r = |\mathbf{r}| \ , \\ \mathbf{v} &\equiv \frac{d\mathbf{r}}{dt} = (\frac{d\xi}{dt}, \frac{d\eta}{dt}, \frac{d\zeta}{dt}) \ , \\ E &= \frac{1}{2} \ \mathbf{v}^2 \ - \ \frac{G \ M_\odot}{r} \ , \\ \mathbf{H} &= \mathbf{r} \times \mathbf{v} \ , \\ p &= \frac{\mathbf{H}^2}{G \ M_\odot} \ , \\ e &= \sqrt{1 \ + \ 2 \ \frac{p \ E}{G \ M_\odot}} \ , \\ a &= \frac{p}{1 - e^2} \ , \end{aligned}$$

$$
\begin{aligned}
E &= -\ \frac{G\ M_\odot}{2\ p}\ \left(1 - e^2\right)\ , \\
q &= a(1-e)\ , \\
i &= \arccos\left(\frac{H_z}{|\mathbf{H}|}\right)\ , \\
\sin\Omega &= \frac{H_\xi}{\sqrt{H_\xi^2 + H_\eta^2}}\ , \\
\cos\Omega &= -\ \frac{H_\eta}{\sqrt{H_\xi^2 + H_\eta^2}}\ , \\
\sin\omega &= \frac{|\mathbf{H}|}{\sqrt{H_\xi^2 + H_\eta^2}}\ \frac{1}{r\ e}\ \times\ S1 \\
S1 &= -\ \frac{\eta\ H_\xi\ -\ \xi\ H_\eta}{|\mathbf{H}|}\ \frac{\mathbf{v}\cdot\mathbf{e}_R}{\sqrt{G\ M_\odot/p}}\ +\ \zeta\ \left(\frac{\mathbf{v}\cdot\mathbf{e}_T}{\sqrt{G\ M_\odot/p}}\ -\ 1\right)\ , \\
\cos\omega &= \frac{|\mathbf{H}|}{\sqrt{H_\xi^2 + H_\eta^2}}\ \frac{1}{r\ e}\ \times\ C1 \\
C1 &= \frac{\eta\ H_\xi\ -\ \xi\ H_\eta}{|\mathbf{H}|}\ \left(\frac{\mathbf{v}\cdot\mathbf{e}_T}{\sqrt{G\ M_\odot/p}}\ -\ 1\right)\ +\ \zeta\ \frac{\mathbf{v}\cdot\mathbf{e}_R}{\sqrt{G\ M_\odot/p}}\ , \\
\mathbf{e}_R &= \frac{\mathbf{r}}{r}\ , \\
\mathbf{e}_N &= \frac{\mathbf{H}}{|\mathbf{H}|}\ , \\
\mathbf{e}_T &= \mathbf{e}_N \times \mathbf{e}_R\ .
\end{aligned}
\tag{26}
$$

4.2 Simple consequence of Eqs. (21)

Eqs. (21) yield

$$
\begin{aligned}
LHS_{S'} &= RHS_{S'}\ , \\
LHS_{S'} &= \frac{d}{dt}\ \left\{\frac{1}{2}\ \left[\left(\frac{d\xi'}{dt}\right)^2\ +\ \left(\frac{d\eta'}{dt}\right)^2\ +\ \left(\frac{d\zeta'}{dt}\right)^2\right]\ -\ \frac{G\ M_\odot}{r}\right. \\
&\quad \left. -\ 2\ A\ (A\ -\ B)\ (\xi')^2\ +\ (2\ \pi\ G\ \varrho\ +\ A^2\ -\ B^2)\ (\zeta')^2\right\}\ , \\
RHS_{S'} &= 2\ (A-B)^2\ (\Gamma_1\ -\ \Gamma_2\ Z_0^2)\ R_0\ Z_0\ \zeta'\ \frac{d\xi'}{dt}\ -\ 4\ \pi\ G\ \varrho'\ Z_0\ \xi'\ \frac{d\zeta'}{dt}\ .
\end{aligned}
\tag{27}
$$

Taking into account

$$
\mathbf{v}' = \mathbf{v}\ -\ \boldsymbol{\omega}\times\mathbf{r}\ , \tag{28}
$$

(Eq. 25 and the relation $\mathbf{r}' = \mathbf{r}$ hold), we have

$$
T' = T\ -\ \boldsymbol{\omega}\cdot(\mathbf{r}\times\mathbf{v})\ +\ \frac{1}{2}\,(\boldsymbol{\omega}\times\mathbf{r})^2\ ,
$$

$$T' \equiv \frac{1}{2} \left[\left(\frac{d\xi'}{dt} \right)^2 + \left(\frac{d\eta'}{dt} \right)^2 + \left(\frac{d\zeta'}{dt} \right)^2 \right] ,$$
$$T \equiv \frac{1}{2} \left[\left(\frac{d\xi}{dt} \right)^2 + \left(\frac{d\eta}{dt} \right)^2 + \left(\frac{d\zeta}{dt} \right)^2 \right] , \tag{29}$$

where T' is the kinetic energy in the rotating noninertial frame of reference and T is the kinetic energy in the inertial frame of reference. Using

$$\begin{aligned} \boldsymbol{\omega} &= - \omega_0 \, \hat{\mathbf{z}} , \\ \hat{\mathbf{z}} \cdot (\mathbf{r} \times \mathbf{v}) &= \hat{\mathbf{z}} \cdot \mathbf{H} \equiv H_z , \\ H_z &= \sqrt{G \, M_\odot \, p} \, \cos i , \end{aligned} \tag{30}$$

Eqs. (26), (27), (29) and (30) yield

$$\begin{aligned} LHS_S &= RHS_S , \\ LHS_S &= \frac{d}{dt} \left\{ - \frac{G \, M_\odot}{2 \, p} \left(1 - e^2 \right) + (A - B) \sqrt{G \, M_\odot \, p} \, \cos i \right. \\ &\quad + \frac{1}{2} (A - B)^2 \left(\xi^2 + \eta^2 \right) \\ &\quad - 2 \, A \, (A - B) \, [\xi \, \cos(\omega_0 \, t) - \eta \, \sin(\omega_0 \, t)]^2 \\ &\quad \left. + (2 \, \pi \, G \, \varrho + A^2 - B^2) \, \zeta^2 \right\} , \\ RHS_S &= 2 \, (A - B)^2 \, (\Gamma_1 - \Gamma_2 \, Z_0^2) \, R_0 \, Z_0 \, \zeta \, \times V \\ &\quad - 4 \, \pi \, G \, \varrho' \, Z_0 \, [\xi \, \cos(\omega_0 \, t) - \eta \, \sin(\omega_0 \, t)] \, \frac{d\zeta}{dt} , \\ V &= \frac{d\xi}{dt} \, \cos(\omega_0 \, t) - \frac{d\eta}{dt} \, \sin(\omega_0 \, t) \\ &\quad - \omega_0 \, \xi \, \sin(\omega_0 \, t) - \omega_0 \, \eta \, \cos(\omega_0 \, t) , \\ \omega_0 &= A - B , \end{aligned} \tag{31}$$

if also equations (following from Eqs. 9)

$$\begin{aligned} \xi' &= \xi \, \cos(\omega_0 \, t) - \eta \, \sin(\omega_0 \, t) , \\ \eta' &= \xi \, \sin(\omega_0 \, t) + \eta \, \cos(\omega_0 \, t) , \\ \zeta' &= \zeta \end{aligned} \tag{32}$$

and equations

$$\begin{aligned} \frac{d\xi'}{dt} &= \frac{d\xi}{dt} \, \cos(\omega_0 \, t) - \frac{d\eta}{dt} \, \sin(\omega_0 \, t) \\ &\quad - \omega_0 \, \xi \, \sin(\omega_0 \, t) - \omega_0 \, \eta \, \cos(\omega_0 \, t) , \\ \frac{d\eta'}{dt} &= \frac{d\xi}{dt} \, \sin(\omega_0 \, t) + \frac{d\eta}{dt} \, \cos(\omega_0 \, t) \\ &\quad + \omega_0 \, \xi \, \cos(\omega_0 \, t) - \omega_0 \, \eta \, \sin(\omega_0 \, t) , \\ \frac{d\zeta'}{dt} &= \frac{d\zeta}{dt} \end{aligned} \tag{33}$$

are used.

Eqs. (31) show that $\Gamma_1 \neq 0$, $\Gamma_2 \neq 0$, $Z_0 \neq 0$ violate the time conservation of the quantity in the curly brackets in the left-hand side of Eq. (31).

5 Galactic tide for Dauphole et al. (1996) model of Galaxy

Dauphole et al. (1996) model of the Galaxy yields [see also Eqs. (29) in Klačka 2009]:

$$
\begin{aligned}
\frac{d^2\xi}{dt^2} &= -\ \frac{GM_\odot}{r^3}\ \xi\ +\ (A-B)\ [A+B+2A\cos{(2\ \omega_0 t)}]\ \xi \\
&\quad -\ 2\ A\ (A-B)\ \sin{(2\ \omega_0 t)}\ \eta \\
&\quad +\ (A-B)^2\ (\Gamma_{1D}/\sqrt{b_d^2+Z_0^2}\ +\ \Gamma_{2D})\ R_0\ Z_0\ \cos{(\omega_0 t)}\ \zeta\ , \\
\frac{d^2\eta}{dt^2} &= -\ \frac{GM_\odot}{r^3}\ \eta\ -\ 2\ A\ (A-B)\ \sin{(2\ \omega_0 t)}\ \xi \\
&\quad +\ (A-B)\ [A\ +\ B\ -\ 2\ A\ \cos{(2\ \omega_0 t)}]\ \eta \\
&\quad -\ (A-B)^2\ (\Gamma_{1D}/\sqrt{b_d^2+Z_0^2}\ +\ \Gamma_{2D})\ R_0\ Z_0\ \sin{(\omega_0 t)}\ \zeta\ , \\
\frac{d^2\zeta}{dt^2} &= -\ \frac{GM_\odot}{r^3}\ \zeta\ -\ \left[4\ \pi\ G\ \varrho\ +\ 2\left(A^2\ -\ B^2\right)\right]\ \zeta \\
&\quad -\ 4\ \pi\ G\ \varrho'\ Z_0\, [\cos{(\omega_0 t)}\ \xi\ -\ \sin{(\omega_0 t)}\ \eta]\ , \\
\frac{d^2 Z_0}{dt^2} &= -\ \left[4\ \pi\ G\ \varrho\ +\ 2\left(A^2\ -\ B^2\right)\right]\ Z_0\ , \\
r &= \sqrt{\xi^2\ +\ \eta^2\ +\ \zeta^2}\ , \\
\omega_0 &= A\ -\ B\ , \\
A &= 14.25\ km\ s^{-1}\ kpc^{-1}\ , \\
B &= -\ 13.89\ km\ s^{-1}\ kpc^{-1}\ , \\
\Gamma_{1D} &= 0.084\ kpc^{-1}\ , \\
\Gamma_{2D} &= 0.008\ kpc^{-2}\ , \\
\varrho &= 0.143\ M_\odot\ pc^{-3}\ , \\
\varrho' &= -\ 0.0425\ M_\odot\ pc^{-3}\ kpc^{-1}\ , \\
b_d &= 0.25\ kpc\ .
\end{aligned}
\tag{34}
$$

Equations analogous to Eqs. (21) are:

$$
\begin{aligned}
\frac{d^2\xi'}{dt^2} &= -\ \frac{GM_\odot}{r^3}\ \xi'\ +\ 4\ A\ (A\ -\ B)\ \xi' \\
&\quad +\ (A-B)^2\ (\Gamma_{1D}/\sqrt{b_d^2+Z_0^2}\ +\ \Gamma_{2D})\ R_0\ Z_0\ \zeta'\ -\ 2\ (A\ -\ B)\ \frac{d\eta'}{dt}\ , \\
\frac{d^2\eta'}{dt^2} &= -\ \frac{GM_\odot}{r^3}\ \eta'\ +\ 2\ (A\ -\ B)\ \frac{d\xi'}{dt}\ , \\
\frac{d^2\zeta'}{dt^2} &= -\ \frac{GM_\odot}{r^3}\ \zeta'\ -\ \left[4\ \pi\ G\ \varrho\ +\ 2\ \left(A^2\ -\ B^2\right)\right]\ \zeta' \\
&\quad -\ 4\ \pi\ G\ \varrho'\ Z_0\ \xi'\ , \\
\frac{d^2 Z_0}{dt^2} &= -\ \left[4\ \pi\ G\ \varrho\ +\ 2\ \left(A^2\ -\ B^2\right)\right]\ Z_0\ , \\
r &= \sqrt{\xi'^{\,2}\ +\ \eta'^{\,2}\ +\ \zeta'^{\,2}}\ , \\
\omega_0 &= A\ -\ B\ .
\end{aligned}
\tag{35}
$$

The vector of the perturbing acceleration acting on the comet is

$$
\begin{aligned}
\mathbf{F}' &= (\mathbf{F}')_{\xi'} \; \hat{\boldsymbol{\xi}}' \; + \; (\mathbf{F}')_{\eta'} \; \hat{\boldsymbol{\eta}}' \; + \; (\mathbf{F}')_{\zeta'} \; \hat{\boldsymbol{\zeta}}' \; , \\
(\mathbf{F}')_{\xi'} &= (A \; - \; B) \; \left\{ 4 \; A \; \xi' \; + \; (A - B) \; \left(\frac{\Gamma_{1D}}{\sqrt{b_d^2 + Z_0^2}} \; + \; \Gamma_{2D} \right) \; R_0 \; Z_0 \; \zeta' \; - \; 2 \; \frac{d\eta'}{dt} \right\} \; , \\
(\mathbf{F}')_{\eta'} &= 2 \; (A \; - \; B) \; \frac{d\xi'}{dt} \; , \\
(\mathbf{F}')_{\zeta'} &= - \; \left[4 \; \pi \; G \; \varrho \; + \; 2 \; \left(A^2 \; - \; B^2 \right) \right] \; \zeta' \; - \; 4 \; \pi \; G \; \varrho' \; Z_0 \; \xi' \; .
\end{aligned}
\tag{36}
$$

Finally, equations analogous to Eqs. (31) are:

$$
\begin{aligned}
LHS_{S \; D} &= RHS_{S \; D} \; , \\
LHS_{S \; D} &= \frac{d}{dt} \; \left\{ - \; \frac{G \; M_{\odot}}{2 \; p} \; \left(1 \; - \; e^2 \right) \; + \; (A - B) \; \sqrt{G \; M_{\odot} \; p} \; \cos i \right. \\
& \quad + \frac{1}{2} \; (A \; - \; B)^2 \; \left(\xi^2 \; + \; \eta^2 \right) \\
& \quad - \; 2 \; A \; (A - B) \; [\xi \; \cos(\omega_0 \; t) \; - \; \eta \; \sin(\omega_0 \; t)]^2 \\
& \quad \left. + \; (2 \; \pi \; G \; \varrho \; + \; A^2 \; - \; B^2) \; \zeta^2 \right\} \; , \\
RHS_{S \; D} &= (A - B)^2 \; (\Gamma_{1D} / \sqrt{b_d^2 + Z_0^2} \; + \; \Gamma_{2D}) \; R_0 \; Z_0 \; \zeta \; \times V \\
& \quad - \; 4 \; \pi \; G \; \varrho' \; Z_0 \; [\xi \; \cos(\omega_0 \; t) \; - \; \eta \; \sin(\omega_0 \; t)] \; \frac{d\zeta}{dt} \; , \\
V &= \frac{d\xi}{dt} \; \cos(\omega_0 \; t) \; - \; \frac{d\eta}{dt} \; \sin(\omega_0 \; t) \\
& \quad - \; \omega_0 \; \xi \; \sin(\omega_0 \; t) \; - \; \omega_0 \; \eta \; \cos(\omega_0 \; t) \; , \\
\omega_0 &= A - B \; .
\end{aligned}
\tag{37}
$$

6 Conclusion

The paper treats the effect of the galactic tide on motion of a comet with respect to the Sun. Eqs. (21) correspond to the relevant equation of motion in the rotating noninertial frame of reference (the $\xi'-$axis is still oriented outward from the galactic center, the $\zeta'-$axis is oriented toward the north Galactic pole and the $\xi'-$, $\eta'-$ and $\zeta'-$ axes form the right-handed coordinate system). The vector of the perturbing force acting on the comet is given by Eqs. (22). The equation of motion produces Eq. (31) which would represent a time independent quantity for the conventionally considered form of the galactic tidal field characterized by the conditions $\Gamma_1 = 0$, $\Gamma_2 = 0$, $Z_0 \equiv 0$ (in reality, the first two conditions or the third condition alone are sufficient).

Solving Eqs. (21) for a comet, one can find an orbital evolution of the comet in terms of orbital elements. System of Eqs. (21), (23)-(24) and (26) has to be used. Sec. 4.1 describes how to transform the solution of Eqs. (21) into the evolution of the osculating orbital elements.

The case of Dauphole et al. model of Galaxy is presented by Eqs. (34)-(37).

Appendix A: Inertial and noninertial frames of reference

(Reference to equation of number (j) of this appendix is denoted as Eq. (A j). Reference to equation of number (i) of the main text is denoted as Eq. (i).)

The text presented between Eqs. (10) and (11) defines the velocity in the noninertial frame of reference (the velocity with respect to the system S') in the form $\mathbf{v}' \equiv d'\mathbf{r}'/dt \equiv (dx'/dt)\ \hat{\mathbf{x}}' + (dy'/dt)\ \hat{\mathbf{y}}' + (dz'/dt)\ \hat{\mathbf{z}}'$. The question "Why is there a prime above the symbol of differentiation in $d'\mathbf{r}'/dt$?" may appear. This appendix explicitly explains the situation.

At first, we have position vectors:

$$\mathbf{r}' = \mathbf{r}\ , \tag{1}$$

$$\mathbf{r}' = x'\ \hat{\mathbf{x}}'\ +\ y'\ \hat{\mathbf{y}}'\ +\ z'\ \hat{\mathbf{z}}'\ , \tag{2}$$

$$\mathbf{r} = x\ \hat{\mathbf{x}}\ +\ y\ \hat{\mathbf{y}}\ +\ z\ \hat{\mathbf{z}}\ . \tag{3}$$

As for the velocity vector, we have

$$\begin{aligned} \mathbf{v} &= \frac{d\mathbf{r}}{dt}\ , \\ \mathbf{v} &= \frac{dx}{dt}\ \hat{\mathbf{x}}\ +\ \frac{dy}{dt}\ \hat{\mathbf{y}}\ +\ \frac{dz}{dt}\ \hat{\mathbf{z}} \end{aligned} \tag{4}$$

in the inertial frame of reference, if Eq. (A3) is used. We can also write, on the basis of Eqs. (A1) and (A2),

$$\begin{aligned} \mathbf{v} &= \frac{d\mathbf{r}}{dt} = \frac{d\mathbf{r}'}{dt} \\ &= \frac{d}{dt}\left(x'\ \hat{\mathbf{x}}'\ +\ y'\ \hat{\mathbf{y}}'\ +\ z'\ \hat{\mathbf{z}}'\right) \\ &= \frac{dx'}{dt}\ \hat{\mathbf{x}}'\ +\ x'\ \frac{d\hat{\mathbf{x}}'}{dt}\ + \\ &\quad \frac{dy'}{dt}\ \hat{\mathbf{y}}'\ +\ y'\ \frac{d\hat{\mathbf{y}}'}{dt}\ + \\ &\quad \frac{dz'}{dt}\ \hat{\mathbf{z}}'\ +\ z'\ \frac{d\hat{\mathbf{z}}'}{dt}\ . \end{aligned} \tag{5}$$

The last relation can be arranged on the basis of Eqs. (7):

$$\begin{aligned} \mathbf{v} = &\ \frac{dx'}{dt}\ \hat{\mathbf{x}}'\ +\ x'\ \{-\ \omega\ \sin(\omega\ t)\ \hat{\mathbf{x}}\ +\ \omega\ \cos(\omega\ t)\ \hat{\mathbf{y}}\}\ + \\ &\ \frac{dy'}{dt}\ \hat{\mathbf{y}}'\ +\ y'\ \{-\ \omega\ \cos(\omega\ t)\ \hat{\mathbf{x}}\ -\ \omega\ \sin(\omega\ t)\ \hat{\mathbf{y}}\}\ + \\ &\ \frac{dz'}{dt}\ \hat{\mathbf{z}}' \\ = &\ \frac{dx'}{dt}\ \hat{\mathbf{x}}'\ +\ \frac{dy'}{dt}\ \hat{\mathbf{y}}'\ +\ \frac{dz'}{dt}\ \hat{\mathbf{z}}'\ + \\ &\ -\ \omega\ \left\{x'\ \sin(\omega\ t)\ +\ y'\ \cos(\omega\ t)\right\}\ \hat{\mathbf{x}} \\ &\ +\ \omega\ \left\{x'\ \cos(\omega\ t)\ -\ y'\ \sin(\omega\ t)\right\}\ \hat{\mathbf{y}}\ . \end{aligned} \tag{6}$$

Using Eqs. (10), we obtain

$$\begin{aligned} \mathbf{v} &= \mathbf{v}' \ - \ \omega \ y \ \hat{\mathbf{x}} \ + \ \omega \ x \ \hat{\mathbf{y}} \\ &= \mathbf{v}' \ + \ \boldsymbol{\omega} \times \mathbf{r}' \ , \end{aligned} \tag{7}$$

since $\mathbf{v}' = (dx'/dt) \ \hat{\mathbf{x}}' + (dy'/dt) \ \hat{\mathbf{y}}' + (dz'/dt) \ \hat{\mathbf{z}}' \equiv d'\mathbf{r}'/dt$.

The last relation of Eq. (A5) can be arranged in an another way. We can use

$$\begin{aligned} \frac{d\hat{\mathbf{x}}'}{dt} &= \boldsymbol{\omega} \ \times \ \hat{\mathbf{x}}' \ , \\ \frac{d\hat{\mathbf{y}}'}{dt} &= \boldsymbol{\omega} \ \times \ \hat{\mathbf{y}}' \ . \end{aligned} \tag{8}$$

Eqs. (A5) and (A8) immediately yield

$$\begin{aligned} \mathbf{v} &= \frac{dx'}{dt} \ \hat{\mathbf{x}}' \ + \ x' \ \boldsymbol{\omega} \ \times \ \hat{\mathbf{x}}' \ + \\ & \quad \frac{dy'}{dt} \ \hat{\mathbf{y}}' \ + \ y' \ \boldsymbol{\omega} \ \times \ \hat{\mathbf{y}}' \ + \\ & \quad \frac{dz'}{dt} \ \hat{\mathbf{z}}' \\ &= \mathbf{v}' \ + \ \boldsymbol{\omega} \times \mathbf{r}' \ . \end{aligned} \tag{9}$$

The final results of Eqs. (A7) and (A9) are equal.

As for acceleration, the results represented by Eqs. (A7) and (A9) yield

$$\begin{aligned} \frac{d\mathbf{v}}{dt} &= \frac{d}{dt} \left(\mathbf{v}' \ + \ \boldsymbol{\omega} \times \mathbf{r}' \right) \\ &= \frac{d\mathbf{v}'}{dt} \ + \ \frac{d}{dt} \left(\boldsymbol{\omega} \times \mathbf{r}' \right) \\ &= \frac{d'\mathbf{v}'}{dt} \ + \ \boldsymbol{\omega} \times \mathbf{v}' \ + \ \boldsymbol{\omega} \times \frac{d\mathbf{r}'}{dt} \\ &= \frac{d'\mathbf{v}'}{dt} \ + \ \boldsymbol{\omega} \times \mathbf{v}' \ + \ \boldsymbol{\omega} \times \left(\mathbf{v}' \ + \ \boldsymbol{\omega} \times \mathbf{r}' \right) \ , \end{aligned} \tag{10}$$

or, simply,

$$\mathbf{a} = \mathbf{a}' \ + \ 2 \ \boldsymbol{\omega} \times \mathbf{v}' \ + \ \boldsymbol{\omega} \times \left(\boldsymbol{\omega} \times \mathbf{r}' \right) \ . \tag{11}$$

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